Agent-based spin model for financial markets on complex networks: Emergence of two-phase phenomena

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We study a microscopic model for financial markets on complex networks, motivated by the dynamics of agents and their structure of interaction. The model consists of interacting agents (spins) with local ferromagnetic coupling and global antiferromagnetic coupling. In order to incorporate more realistic situations, we also introduce an external field which changes in time. From numerical simulations, we find that the model shows two-phase phenomena. When the local ferromagnetic interaction is balanced with the global antiferromagnetic interaction, the resulting return distribution satisfies a power law having a single peak at zero values of return, which corresponds to the market equilibrium phase. On the other hand, if local ferromagnetic interaction is dominant, then the return distribution becomes double peaked at nonzero values of return, which characterizes the out-of-equilibrium phase. On random networks, the crossover between two phases comes from the competition between two different interactions. However, on scale-free networks, not only the competition between the different interactions but also the heterogeneity of underlying topology causes the two-phase phenomena. Possible relationships between the critical phenomena of spin system and the two-phase phenomena are discussed.

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I. INTRODUCTION

Power-law or fat-tailed distributions observed in many economic systems [1,2] have attracted many physicists because of their relevance to critical phenomena in statistical physics. Especially, empirical studies on logarithmic price changes (returns) in real markets have found the intermittent occurrence of large bursts resulting in the power-law tails in their distributions. Various models have been introduced to understand the origin of the observed properties in real financial markets [2–7]. Among these models, Ising-like spin systems have been studied by some researchers, due to their simplicity [5–7]. For example, Chowdhury and Stauffer [5] introduced a super-spin and time-dependent individual bias of each agent to show that the return distribution caused by herding of agents satisfies a power-law scaling. The effects of the heterogeneity of interaction topology on herding behavior is also studied, which is closely related to the fat-tailed distribution of return [8,9].

Recently, more realistic models have been introduced [6–8,10,11], in which the pool of agents is divided into two groups: "Fundamentalist" (or "chartist") and "trend follower" (or "noise trader"). The fundamentalists are those who exactly know the excess demand (difference between the demand and supply), and the trend followers are those who follow the decision of their interacting neighbors. The coexistence of the fundamentalist and the trend follower contradicts the prevalent "efficient market hypothesis" in economics. These models succeeded in reproducing several nontrivial properties in a real market. For example, Sznajd-Weron and Weron studied the effects of a single fundamentalist by using Ising-like ferromagnetic interactions on two sublattices [7]. Another interesting spin model which incorporates the competition between the fundamentalist and trend follower was suggested by Bornholdt [6]. In Bornholdt's model, each agent is assumed to have features of both the fundamentalist and the trend follower. The ferromagnetic interaction with nearest neighbors is used to stand for the characteristics of the trend followers. At the same time, each spin (or agent) interacts with the global magnetization. The global interaction represents the tendency of the fundamentalist by encouraging a spin-flip when the global magnetization becomes large. The global interaction depends only on the magnitude of the magnetization, but not the current state of each agent. Thus, the global interaction in Bornholdt's model implicitly contains the effective time-dependent bias of each spin, which is similar to that of the Chowdhury and Stauffer model [5].

In order to study the origin of the power-law distribution of the return, we have recently investigated a spin model on a two-dimensional square lattice based on the microscopic dynamics of each agent [12]. The model has the explicit time-dependent global field as well as the competition between the characteristics of noise trader and the fundamentalist. The time-dependent global field represents any internal or external interferences in the market dynamics. By numerical simulations, we have shown that the competition between trend follower and fundamentalist causes two different domains if the temperature is lower than the critical temperature of Ising model, below which the system is in the ordered state. When this domain structure is completely destroyed by the external field and restored, we find large bursts in the return to make a power-law or fat-tailed distribution of return.

In this paper, since all the agents in a real market do not have the same number of interacting partners, we study the effect of the heterogenous structure of interactions between each agent on the market dynamics. Moreover, the critical behavior of the Ising model on complex networks is known

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to be different from that on regular structures. Especially, on the scale-free networks (SFN), in which the degree distribution follows a power law, $P(k)\sim k^{-\gamma}$ [13–15], the critical behavior of Ising model strongly depends on γ. For $\gamma<3$ the critical temperature, T_c^0, of the simple Ising model goes to infinity as $N\to\infty$. On the other hand, for $3<\gamma<5$ the system shows second-order phase transition but the obtained critical exponents depend on γ. When $\gamma>5$ the Ising model satisfies the mean-field expectation. Therefore, it is also theoretically interesting to study how these complicated critical behavior of spin models affect the market dynamics. By numerical simulations we show that the heterogeneity of the underlying structure causes two-phase phenomena [16]. For the theoretical considerations, we use two network topologies, random network (RN) [17] and static SFN [15]. The paper is organized as follows: In Sec. II, we introduce our model. Numerical results and some possible relations between the observed two-phase phenomena and the critical phenomena of spin system are presented in Sec. III. Finally, the summary and discussions will be given in Sec. IV.

II. MODEL

We consider N agents placed on each node in a given network. The state of each agent i is characterized by a two-state spin variable $s_i(t)\in\{-1,+1\}$ at time t. Each state corresponds to buying (+1) or selling (−1) state. The state of each agent i at time $t+1$, $s_i(t+1)$, is updated by the heat-bath dynamics [18] based on the Hamiltonian

$$H(t) = -\sum_{i>j}^{N} J_{ij}s_i(t)s_j(t) + [\alpha M(t) - f(t)]\sum_{i=1}^{N} s_i(t). \quad (1)$$

Here, $J_{ij}=J(>0)$ if i and j are nearest neighbors (ferromagnetic interaction); otherwise, $J_{ij}=0$. For simplicity, we let the Boltzmann constant k_B and J be unity. This ferromagnetic interaction represents the tendency that each agent follows the decision of his cooperating agents (characteristics of the trend follower). In the second term, $M(t)=(1/N)\Sigma_j^N s_j(t)$ corresponds to the excess demand. For $\alpha>0$, the interaction becomes antiferromagnetic and stands for the tendency of the fundamentalist, who exactly knows the excess demand. The antiferromagnetic interaction reflects the following fact; if the demand (supply) exceeds the supply (demand), then each agent wants to place a selling (buying) order to maximize his benefit. $f(t)$ denotes a time-dependent external field which incorporates all internal and external interference in the market dynamics. In general, the magnitude of this interference changes in time. Furthermore, large interferences such as oil shock and subprime mortgage crisis do not have the same occurrence probability as daily reported rumors. Thus, at each time step we choose the magnitude of the external field, $|f(t)|$, from the power-law distribution

$$P_f(|f|) \sim \frac{1}{|f|^{\sigma}}, \quad (2)$$

where σ determines the heterogeneity of the distribution. The sign of $f(t)$ is chosen at random. The unit time step is defined by the usual Monte Carlo time step.

Before discussing the results on complex networks, let us briefly define the price and return. For simplicity, we assume that the evolution of price, $p(t)$, follows [19]

$$\frac{dp(t)}{dt} = cM(t)p(t), \quad (3)$$

where c is a scaling factor for price change. For small time interval Δt, we obtain $p(t+\Delta t)=p(t)\exp[cM(t)\Delta t]$ by assuming that the change of $M(t)$ during this interval is negligible. In a discrete time step $(\Delta t=1)$, $p(t+1)=p(t)\exp[cM(t)]$. Thus, the logarithmic return becomes

$$R(t) = \ln p(t) - \ln p(t-1) = cM(t-1). \quad (4)$$

III. RESULTS

In simulations, we use networks with $N=10^4$ nodes. We find that our main conclusion does not crucially depend on N.

A. Random networks

RN is generated by connecting each pair of nodes with a fixed probability ϕ [17]. By adjusting ϕ, we fix the average degree to be $\langle k\rangle=4$.

If $T>T_c^0$ or $\alpha\gg\langle k\rangle$, then the system is always in the disordered phase, and $P(R)$ trivially depends only on $P(f)$. Here T_c^0 means the critical temperature for the simple Ising model or H with $\alpha=f(t)=0$ in Eq. (1). When $T=3.0$ which is close to (but smaller than) T_c^0 (≈ 4 for $\langle k\rangle=4$) and $\alpha\approx\langle k\rangle$, the domainlike structure is formed as a result of the competition between ferromagnetic and antiferromagnetic interactions. For these values of parameters, the systems is in the market equilibrium phase as shown in Figs. 1(a) and 1(b). Figure 1(a) shows the time evolution of $R(t)$ when $\sigma=2.5$, $\alpha=4$, and $T=3.0$. Since the average degree of the network is $\langle k\rangle=4$, the antiferromagnetic interaction becomes comparable with the ferromagnetic interaction when $T=3(<T_c^0)$ and $\alpha\approx 4$. With these values of parameters, we find the intermittent occurrences of large bursts in $R(t)$, which are very similar to those of real market indices [20] and the results on two-dimensional square lattices [12].

In Fig. 1(b) we display $P(R)$ for the same set of parameters with the data in Fig. 1(a). In general, $P(R)$ of the real market has been approximated by a Lévy stable distribution [2,12,21]:

$$P(R) \equiv \frac{1}{\pi}\int_0^{\infty} \exp(-a|q|^{\mu})\cos(qR)dq. \quad (5)$$

Here, μ and a are the Lévy exponent and scaling factor, respectively. For large $|R|$, Eq. (5) can be written as a power law,

$$P(|R|) \sim |R|^{-(1+\mu)}. \quad (6)$$

From the best fit of Eq. (6) to the data in Fig. 1(b), we find $\mu=1.5\pm 0.1$ for $|R|>2$. The solid line in Fig. 1(b) represents Eq. (5) with the obtained $\mu(\simeq 1.5)$. For convenience we use $c=240$ on RNs. The other choice of c affects only on a

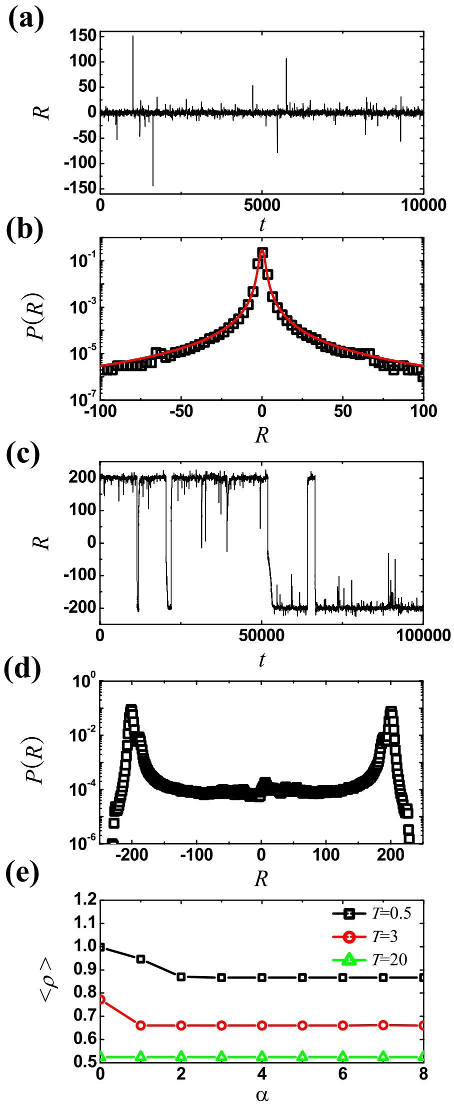

FIG. 1. (Color online) (a) Time evolution of $R(t)$ on RN and (b) plot of $P(R)$ when $T=3.0$ (near T_c^0), $\alpha=4$, and $\sigma=2.5$. The solid line represents Eq. (5) with $\mu=1.5$. (c) Plot of $R(t)$ and (d) $P(R)$ for $T=0.5$, $\alpha=1$, and $\sigma=2.5$. (e) Segregation function for various T when $f(t)=0$.

without changing μ. The result shows that $P(R)$ obtained from the simulations agrees very well with Eq. (5). For this case, the market is in the equilibrium phase in which the price is determined by nearly the same number of buyers and sellers [16]. As σ increases, $P_f(|f|)$ becomes relatively homogeneous. Thus, the price fluctuations are not correlated and $P(R)$ becomes Gaussian.

When T is very low and α is small, $R(t)$ does not show any intermittent bursts [see Fig. 1(c)]. Thus, $P(R)$ does not satisfy the power-law distribution as shown in Fig. 1(d). Since the ferromagnetic interaction is always dominant for this value of T and α ($<\langle k\rangle$), the system is always in the ferromagnetic ordered state when $|f|=0$. The ferromagnetic ordered state indicates that there is an imbalance in the number of buyers and sellers. The imbalanced state is relatively stable against the change of external field. As a result, $R(t)$ changes its sign only when $|f(t)|$ is large enough. In this case,

TABLE I. Market behavior for various values of α, σ, and T ($<T_c^0$) on the random networks. ×'s in the table indicate that the distribution is single peaked at $R=0$, but it follows neither a power law (Lévy distribution) nor a Gaussian distribution. For $\mu>2$, the Lévy distribution is not stable.

	σ	$T=0.5$	$T=3.0$
	2.5	Double peak	×
$\alpha=1$	3.0	Double peak	×
	3.5	Double peak	Gaussian
	2.5	Lévy ($\mu=1.9\pm0.1$)	Lévy ($\mu=1.5\pm0.1$)
$\alpha=4$	3.0	Lévy ($\mu=2.8\pm0.1$)	Lévy ($\mu=2.3\pm0.1$)
	3.5	Gaussian	Gaussian

the market behavior is governed by buyers for one-half of the time, and by sellers for the other one-half as shown in Fig. 1(c). Accordingly large fluctuations in return emerge. Therefore, the market is in the out-of-equilibrium phase [16] in which $P(R)$ is double-peaked at nonzero R [see Fig. 1(d)]. Other results for various values of parameters are summarized in Table I.

When the ferromagnetic and antiferromagnetic interactions are balanced and $P(f)$ has relatively high heterogeneity ($\sigma<3$), the Lévy stable distribution is obtained for $T<T_c^0$. The similar results were reported for the model on two-dimensional square lattices [12]. The formation of two different domains on a two-dimensional square lattice is easily verified by taking a snapshot when $|f|=0$ [12]. However, analyzing the snapshot does not give us any information in complex networks due to their heterogeneous topologies. In order to avoid this difficulty, we use the segregation function [22] for the detailed investigation. The segregation function of node i in state s_i is defined as

$$\rho(s_i)=\frac{n(s_i)}{k_i}, \tag{7}$$

where $n(s_i)$ denotes the number of i's nearest neighbors which have the same state with i, and k_i is the degree of i. The average segregation function $\langle\rho\rangle$ is obtained by averaging $\rho(s_i)$ over all nodes in the network. If all nodes are in the same state, then $\langle\rho\rangle$ becomes 1. If $T>T_c^0$, then $\langle\rho\rangle\to 1/2$ in the limit $N\to\infty$. If different domains are formed, then $\langle\rho\rangle$ lies between 1/2 and 1. When T is very low, $\langle\rho\rangle$ is $0.9\sim1$ for $\alpha\leqslant 2$ [see Fig. 1(e)]. This indicates that the system is in the ferromagnetic ordered state to make the out-of-equilibrium phase when $T=0.5$ and $\alpha=1$ [see Fig. 1(d)]. When $T=3.0$ and $\langle k\rangle/2\leqslant\alpha\leqslant 2\langle k\rangle$, $\langle\rho\rangle$ lies between 0.6–0.8. In this regime of parameters, the system is in the market equilibrium phase as shown in Figs. 1(a) and 1(b).

B. Scale-free networks

The scale-free network (SFN) is the network whose degree distribution satisfies a power law, $P(k)\sim k^{-\gamma}$. Recent studies have found that the critical phenomena of the simple Ising model on SFNs with small $\gamma<5$ are different from

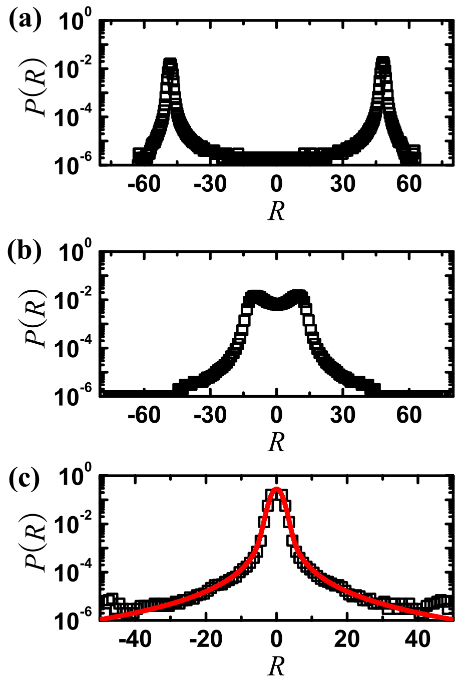

FIG. 2. (Color online) Plot of $P(R)$ on SFN when $\sigma=2.5$ and $\gamma=3.5$ with (a) $\alpha=1$ and $T=1$, (b) $\alpha=4$ and $T=1$, and (c) $\alpha=4$ and $T=4$ which is relatively close to T_c^0 (≈ 6). The solid line in (c) represents Eq. (5) with $\mu=1.9$.

those of mean-field expectations [23,24]. The critical exponents obtained from the local tree approximation depend on the degree exponent γ. This indicates that the heterogeneity of the underlying structure affects the critical behavior of the Ising model. Moreover, many social and economic systems are known to form various SFNs [25]. Therefore, it has theoretical and practical importance to investigate how the heterogeneity of the underlying topology affects the market dynamics. For this purpose, we use a static model suggested by Goh *et al.* [15] to generate SFN with tunable γ. The average degree $\langle k \rangle$ is fixed to be 4 for the direct comparison with the results on RN and two-dimensional lattices [12]. Since the size N of network does not affect the conclusions, we present the results only for $N=10^4$. In order to demonstrate our results in an appropriate price scale, we use $c=140$ when T is very low and $c=70$ when T is close to T_c^0. The main conclusions also do not depend on the value of c.

We find that our model on SFNs also shows a two-phase phenomena. For $\gamma>5$, we find nearly the same results with RNs. However, the origin of the observed crossover from market equilibrium phase to out-of-equilibrium phase is different from that on RNs when $\gamma<5$. In the following sections, we discuss the differences in detail for $\gamma<5$.

C. $3<\gamma<5$

In Fig. 2(a) we show $P(R)$ when $\alpha=1$ ($<\langle k \rangle$) and $T=1.0$ on SFN with $\gamma=3.5$. For these values of α and T the ferromagnetic interaction becomes dominant as in the case on RNs and SFNs with $\gamma>5$. The market is thus in the out-of-equilibrium phase, and $P(R)$ becomes double peaked when $f(t)\neq 0$.

TABLE II. Market behaviors for various values of α, σ, and T ($<T_c^0$) on SFNs with $3<\gamma<5$. For $\mu>2$ the Lévy distribution is not stable.

	σ	$T=1.0$	$T=4.0$
	2.5	Double peak	Double peak
$\alpha=1$	3.0	Double peak	Double peak
	3.5	Double peak	Double peak
	2.5	Double peak	Lévy ($\mu=1.9\pm0.1$)
$\alpha=4$	3.0	Double peak	Lévy ($\mu=3.4\pm0.1$)
	3.5	Double peak	Gaussian

At low T or $T=1.0$, the market is also in the out-of-equilibrium phase even for $\alpha\approx\langle k \rangle$ [see Fig. 2(b)]. It indicates that not only the competition between ferromagnetic and antiferromagnetic interactions but also the heterogeneity of underlying topology causes the two-phase phenomena. This results for $3<\gamma<5$ is quite different from that on RNs and SFNs with $\gamma>5$, where the market is in the equilibrium phase for low T and $\alpha\approx\langle k \rangle$. From the numerical simulations, we find that the system is always in the ferromagnetic ordered state up to $\alpha\approx\langle k \rangle$ when $T<T_c^0$, $\gamma<5$, and $|f|=0$ (which is not shown). Since the susceptibility of the simple Ising model decays as $\chi\sim|T_c^0-T|^{-1}$ for $3<\gamma<5$ [23], this ferromagnetic ordered state becomes more stable as $|T_c^0-T|$ increases. As a result, $M(t)$ or equivalently $R(t)$ changes its sign only when the occasional strong external field is applied and the fluctuation in the market becomes large. Thus, the market is in the out-of-equilibrium phase in which the market behavior is governed by buyers for one-half of the time and by sellers for the other one-half depending on the sign of the applied external field even for $\alpha\approx\langle k \rangle$ [see Fig. 2(b)]. This result is consistent with the previous expectation that the system is in the out-of-equilibrium phase when the market fluctuation or the local noise intensity becomes large [16].

When T is close to $T_c^0(\approx 6)$ or $T=4$ and $\alpha=1(<\langle k \rangle)$, the ferromagnetic interaction is dominant. Moreover, for $3<\gamma<5$, $|M|$ of simple Ising model increases more rapidly as $|T_c^0-T|$ increases than that on RNs or SFNs with $\gamma>5$ [23]. This causes a large fluctuations in $M(t)$ or $R(t)$ when $|f(t)|$ is large enough. Thus, the market is still in the out-of-equilibrium phase (see the case of $T=4$ and $\alpha=1$ in Table II). But when T is close to T_c^0 (or $T=4$) and $\alpha\approx\langle k \rangle$, the market is in the equilibrium phase with single peaked $P(R)$ as shown in Fig. 2(c). When T is close to T_c^0 and $\alpha\approx\langle k \rangle$, $P(R)$ becomes a Gaussian distribution for large σ (see Table II).

D. $2<\gamma<3$

As shown in Fig. 3(a), $P(R)$ is double peaked which corresponds to the out-of-equilibrium phase when $T=1$ and $\alpha=4$.

Due to the heterogeneity of the underlying networks, the critical behavior of the Ising model on SFN with $\gamma<3$ becomes completely different from that observed on RN or SFN with $\gamma>3$. In this regime, $T_c^0\to\infty$ in the limit $N\to\infty$

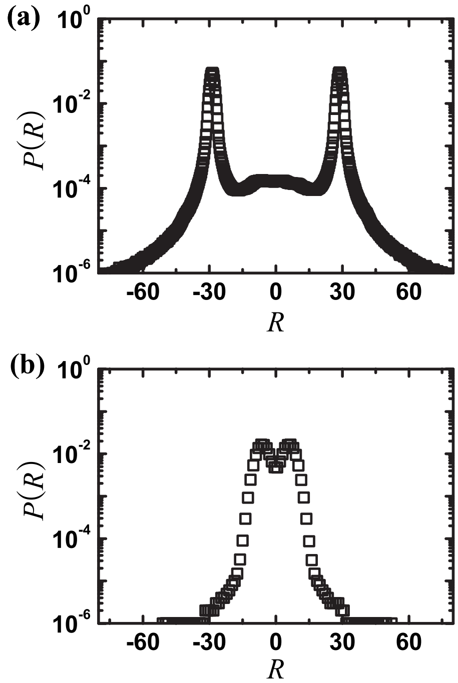

FIG. 3. Plot of $P(R)$ on SFN with $\sigma=2.5$, $\alpha=4$, and $\gamma=2.5$ when (a) $T=1$, and (b) $T=16$ which is close to T_c^0 (≈ 25 for $N=10^4$).

[23] and the susceptibility decreases as $\chi \sim 1/T$. Although T_c^0 for $2<\gamma<3$ diverges, we can determine the $T_c^0(N)$ for the finite-sized network with the size N. [23]. The small susceptibility near $T_c^0(N)$ and the imbalance in the number of buyers and sellers cause very stable domains against the change of external field when $2<\gamma<3$. This stable domain can be disturbed only when the occasional large $|f|$ is applied. In Fig. 3(b) we show that the $P(R)$ is double-peaked at nonzero R when T is close to $T_c^0(N)$ (or $T=16$) and $\alpha=4$. For $T=16$ and $\alpha=50$, double peak in $P(R)$ still remains (which is not shown). Since $T_c^0 \to \infty$ in the limit $N\to\infty$, the antiferromagnetic interaction is balanced with the ferromagnetic interaction only when $\alpha\to\infty$. Thus, the system is always in the ferromagnetic ordered state for any finite α. Therefore, the market is always in the out-of-equilibrium phase in the limit $N\to\infty$. Results for other values of parameters are shown in Table III.

TABLE III. Market behaviors for various values of α, σ, and T ($<T_c^0$) on SFNs with $2<\gamma<3$.

	σ	$T=1.0$	$T=16$
	2.5	Double peak	Double peak
$\alpha=1$	3.0	Double peak	Double peak
	3.5	Double peak	Double peak
	2.5	Double peak	Double peak
$\alpha=4$	3.0	Double peak	Double peak
	3.5	Double peak	Double peak

IV. SUMMARY AND DISCUSSION

We study a generalized spin model for price changes in a financial market on complex networks motivated by the characteristics of agents. The model assumes that each agent has tendencies of both trend follower and fundamentalist. Each tendency is represented by ferromagnetic interaction with nearest neighbors and antiferromagnetic interaction with a global self-generated field. The antiferromagnetic interaction assumes that each agent is smart enough to make his decision to be a minority in the market dynamics. From the numerical simulations we find that there exists two-phase phenomena on complex networks [16]. When the ferromagnetic interaction is dominant, $P(R)$ becomes double peaked. On the other hand, if the number of sellers balances with the number of buyers, then $P(R)$ becomes usual fat-tailed distribution.

These results provide a clue to understanding the market dynamics. Our model indicates that the balance between sellers and buyers can be achieved by competition between the fundamentalist's feature (global antiferromagnetic interaction) and the characteristics of the trend follower (ferromagnetic interaction) to maximize the benefit of each agent. If large market interference is applied to this balanced state, then most of the buyers (sellers) are abruptly changed to sellers (buyers). The sudden changes between buyers and sellers cause intermittent occurrence of large bursts depending on the nature of the market interference distribution and underlying topologies. Moreover, when the fluctuation in the number of buyers or sellers in the market is small, $P(R)$ can be approximated by Eq. (5) which is the typical feature of the market equilibrium phase. On the other hand, if the market fluctuation becomes large then the $P(R)$ becomes double peaked. Thus the market undergoes a crossover from market equilibrium phase to out-of-equilibrium phase. The results shows a good agreement with other empirical analysis [16]. We also find that on RNs and SFNs with $\gamma>5$ such crossover is caused by the temperature and the competition between ferromagnetic and antiferromagnetic interactions. However, as we decrease γ of SFNs, the underlying topology also affect the crossover between the two phases. Furthermore, when $2<\gamma<3$ and $T\lesssim T_c^0$ we find that the market is always in the out-of-equilibrium phase.

Finally, the fluctuation of inverse concentration of buyers (or sellers) was studied in the agent-based herding model, and it was found that RNs can cause fat-tailed distribution through the mean-field approach [8]. In our spin model, we find that not only the spin fluctuations but also fluctuation of underlying topology can cause many interesting behaviors, such as two-phase phenomena.

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